Numerical simulation of accretion discs in close binary systems and discovery of spiral shocks

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Abstract. The history of hydrodynamic numerical simulations for accretion disks in close binary systems is reviewed, in which emphasis is placed, in particular, on the facts that spiral shock waves were numerically found in 1986 by researchers including one of the present authors and that spiral structure was discovered in IP Pegasi in 1997 by Steeghs et al. The results of our two and three-dimensional numerical simulations in recent years are then summarized, with comparison being made with observations.

Keywords: Accretion disk, numerical simulation, spiral shock, IP Pegasi, Doppler map

1. Introduction

The term "accretion" used in astrophysics means the infall of gas onto a celestial body due to the gravitational attraction of the body. On this occasion, the gravitational energy of the gas is released and, eventually, transformed into radiation and emitted to space.

The accretion process may be classified into two types depending on the momentum possessed by the gas, i.e. whether the gas has a large angular momentum with respect to the accreting object or not. In the former case, a thin gaseous disk, "accretion disk", is formed around the accreting object. In such a disk, the gas gradually loses its angular momentum by some mechanism and accretes onto the central object. On the other hand, a gas with less angular momentum rapidly accretes onto the object directly and this is called "wind accretion", a process characterized by, when the wind is supersonic, the formation of "bow shock" in the forepart of the accreting object. The present paper deals with accretion disks.

Accretion disks are further classified into those in which the accreting body has a mass comparable to that of typical star and those in which the accreting body is a giant black hole expected to be present at the central core of galaxy. Our subject in this paper relates to the former. With the mass-accreting star being what is known as a compact star, which has a very small size compared to its mass, such as black hole, neutron star or white dwarf, a large amount of energy is released on accretion. Such accretion disks are present in close binaries, including cataclysmic variables, novae and X-ray stars.

In accretion disks, for gas to accrete onto the central body, the gas must lose, by some mechanism or another, its angular momentum. The standard model proposed by Shakura & Sunyaev (1973) states that the angular momentum is transported from the inner parts of the disk to the outer parts, due to some kind of viscosity (see also Shakura 1972a, b, Pringle & Rees 1972). The gas of the inner parts, having lost angular momentum, accretes onto the central star, while a fraction of the outer gas, having obtained larger angular momentum, transports it to infinity. The gas can thus, almost entirely, accrete, while conserving the total angular momentum.

A generally accepted candidate for this viscosity is turbulent viscosity. The standard model is often called the "α-disk model", since the magnitude of the turbulent viscosity has been characrerized by a phenomenological parameter, α. For accretion disks, the Reynolds number is as large as 10^{11-14}. The usual hydrodynamical common sense expects a flow with such a large Reynolds number to be turbulent. With a rotational fluid, however, the situation is not so simple. Keplerian disks, with angular momentum increasing outwards, amply satisfy the Rayleigh criterion of stability. That is, where the gas rotates with nearly Keplerian motion, there is no convincing evidence so far for the accretion disk to become unstable. In contrast, there is available theoretical and numerical evidence of the stability of Keplerian disks (Balbus, Hawley & Stone, 1996). Balbus & Hawley (1991) concludes that a small magnetic field present in the disk will be amplified during differential rotation and, eventually, generate magnetic turbulence. This has become one of the most popular models in recent years.

Apart from the standard model, Sawada, Matsuda & Hachisu (1986a, b, 1987) conducted two-dimensional numerical simulations of accretion disk, to discover the presence of spiral shock waves in the disk. Spruit (1987) found self-similar solutions having spiral shock waves. They proposed a model in which the spiral shock waves absorb angular momentum from the gas. This is, eventually, transported to the orbital angular momentum of the binary system, due to the axial asymmetry of the density distribution in the disk of the gas and to the torque caused by the tidal force of the companion star. We call this the "spiral shock model", and we describe it in detail in the present paper. Many textbooks and reviews of accretion disks are available (for example, Pringle 1981, Frank, King & Raine 1992, Spruit 1995, Hartmann 1998, Kato, Fukue & Mineshige 1998).

2. Historical overview of numerical simulation

2.1. TWO-DIMENSIONAL HYDRODYNAMIC SIMULATION OF ACCRETION DISK

The particle model was mainly used in early stages of numerical study for accretion disks in close binaries. Particles differ from fluid elements in that the trajectories of the former can cross each other, while those of the latter cannot. Prendergast (1960) was the first to carry out numerical simulation of gaseous flows, while ignoring the pressure. His model expressed the two constituent stars as two mass points and ignored either release or accretion of the gas (see also Huang 1965, 1966). These drawbacks were corrected by Prendergast & Taam (1974), who used the beam scheme and, with the mass-accreting star having a large size, were unable to find formation of any accretion disk. Biermann (1971) conducted simulation by the characteristic line method, for models which were close to wind accretion rather than accretion disks.

Sørensen, Matsuda & Sakurai (1974, 1975) made calculations using the Fluid in Cell method (FLIC) and Cartesian coordinates. They took both the mass-losing star and the mass-accreting star into consideration and assumed the latter to be of such a sufficiently small size as to allow formation of an accretion disk. The results of their calculation showed a gas stream from the L1 point towards the compact star and formation of an accretion disk. Flannery (1975) performed a similar calculation and suggested the presence of a hot spot.

In contrast to the finite-difference method used by them, Lin & Pringle (1976) and Hensler (1982) performed calculations with a particle method, the former in particular using the sticky particle method, which can be called a predecessor of the SPH method. All these calculations, having incorporated an artificial viscosity to stabilize the calculation, could not reveal the detailed structure of the inside of an accretion disk.

Eleven years after the Sørensen et al. (1975), Sawada, Matsuda & Hachisu (1986a, b, 1987) made the second challenge to the same problem, with use of such state-of-the-art techniques as the Osher upwind finite-difference method with 2nd order accuracy, generalized curvilinear coordinates and a super computer of vector type. The Osher upwind difference method can run the calculation stably while suppressing the artificial viscosity at a low level and is a predecessor of the TVD method, which is a representative modern computational fluid dynamics scheme. As a result, they discovered in the accretion disk the presence of spiral shocks—the very feature having been never discovered with use of other more dissipative schemes.

Since then, various authors have been carrying out two-dimensional simulations for accretion disks by various methods and they all obtained spiral shocks (Spruit et al. 1987, Rozyczka & Spruit 1989, Matsuda et al. 1990, Savonije, Papaloizou & Lin 1994, Godon 1997).

Figure 1 shows the results of the two-dimensional simulation performed recently by Makita, Miyawaki & Matsuda (1998). The mass ratio of the binary is 1. The region of calculation, which is limited to the surroundings of the mass-accreting star, is in the range $[-0.5a, 0.5a] \times [-0.5a, 0.5a]$, where a denotes the separation of the binary. The number of grid points is 200×200. Calculation is made for the specific heat ratio, γ, of 1.01, 1.05, 1.1 and 1.2, while using the equation of state for a perfect gas. The lower γ is used to take somehow cooling effects into account.

The gas is assumed to flow into through a hole placed at the L1 point. The density of the gas in the hole is 1 and the sonic velocity is $0.1a\Omega$, where Ω is the angular velocity of rotation. Specifying the sonic velocity means specifying the gas temperature. The sonic velocity used in this calculation is very large, thus indicating that the gas has a considerably high temperature. The gas ejected through the hole expands into a surrounding atmosphere having high temperature, low density and low pressure, thereby forming so called "under-expanded jet". This results in the strange form of the inflow from the L1 point, which phenomenon turns out, however, not to be a serious drawback according to our later study.

What the calculations revealed is that the spiral shocks are generated in any case and that the pitch angle of the spiral arms has a clear correlation with the specific heat ratio. A smaller specific heat ratio thus leads to a tighter winding-in angle of the resulting shock waves. This is because that the smaller specific heat ratio causes lower gas temperature and hence smaller sonic velocity, whereby the spiral shock waves having been generated in the peripheral region wind in more tightly while propagating into the inner parts. In three-dimensional calculations to be described later herein, the problem is that such a clear correlation is not seen.

2.2. Spiral shocks in the universe

The presence of spiral shocks was discovered by Sawada, Matsuda & Hachisu(1986a, b, 1987) in accretion disks. Spiral structures themselves

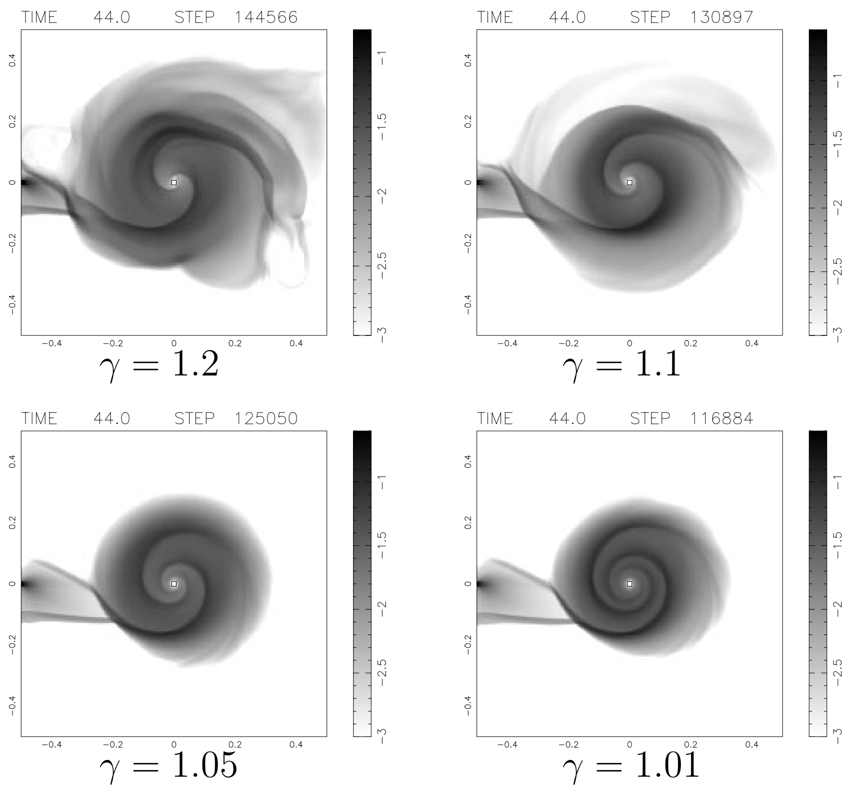

Figure 1. Density distribution in two-dimensional calculation. Logarithmic density of gas is shown at t=44, corresponding to about 7 revolution periods, for four cases of γ, *i.e.* 1.2, 1.1, 1.05, 1.01. Bar on the right side shows the scale range.

are, however, rather common phenomena in astrophysics. In particular, spiral arms of galaxies attracted much attention in the 60-70's. Lin & Shu (1964) contended that spiral density waves are formed in galactic disks under influence of self-gravity. This is the famous density wave theory of spiral arms. Fujimoto (1968) suggested that the spiral arms, actually shining, is a gas rather than the constituent stars and that spiral shock waves composed of the gas have been formed. Shu, Milione & Roberts (1973) discussed the motion of gas in the *spiral gravitational potential* formed by stars and showed that the spiral gravitational potential with its amplitude exceeding a specific level causes the gas to form spiral shock waves.

Sørensen, Matsuda & Fujimoto (1976) conducted numerical simulations for gas flow in a barred galaxy and showed that shock waves had been formed both inside and outside the corotation radius. The outside shock waves are spiral and correspond to the spiral arms, while the inside shocks are linear and correspond to the dark lane where cosmic dust has gathered.

Sørensen & Matsuda (1982) and Matsuda et al. (1987) showed that a barred galaxy with a small deviation from axisymmetry of gravitational potential, i.e. a weak bar, causes the shock waves inside the corotation radius also to become of spiral form. They investigated the correlation between the presence of spirals and that of Lindblad resonance, and concluded that the Lindblad resonance is essential to generate spirals. An important fact is that spiral shocks are generated by the *barred gravitational potential* rather than the *spiral gravitational potential*. This mechanism is essentially the same as that for the formation of the spiral structure in accretion disks of close binaries discussed in the present paper.

Matsuda & Nelson (1977) suggested that the presence of, if any, a weak bar structure at the center of our Galaxy would cause spiral shock waves to generate, whereby gas around the waves loses its angular momentum and energy and falls towards the center. They called this mechanism "vacuum cleaner". This mechanism is important in considering gas supply to the central core in AGN.

Besides the galaxy scale, formation of spiral structure due to tidal force is seen on various objects. For instance, various simulations have shown, with the primordial solar nebula after formation of Jupiter, appearance of spiral shock waves in the gaseous disk. These spiral shocks remove gases from the primordial solar nebula eventually.

In essence spiral shock waves are generated in an accretion disk by an oval deformation of the gravitational potential and the Lindblad resonances associating with it. We stress that spiral shocks are not formed by the collision of the stream from L1 point with the disk gas.

2.3. Does a spiral shock appear on three-dimensional scale?

As described above, the presence of spiral shocks in two-dimensional accretion disk has been verified by various simulations. For three-dimensional disks, it has been argued that waves once formed in the periphery of a disk diffract upwards and do not move into the inside of the disk and that, as a result, spiral shocks cannot appear in the disk (Lin, Papaloizou & Savonije 1990a, b, Lubow & Pringle 1993).

As regards numerical simulation, Molteni, Belvedere & Lanzafame (1991) and Lanzafame, Belvedere & Molteni (1992) conducted three-dimensional simulations by the SPH method and could not find any spiral shock. They further argued that, with the specific heat ratio appearing in the equation of state, γ, being larger than 1.1-1.2, accretion disk itself cannot be formed. Attention should however be paid to the fact that they used in their calculation particles in as small a number as 1159 ($\gamma = 1.2$) or 9899 ($\gamma = 1.01$). Since then for some time, many simulations have been performed mainly with use of the SPH method (Hirose, Osaki & Mineshige 1991, Nagasawa, Matsuda & Kuwahara 1991, Lanzafame, Belvedere & Molteni 1993), but none of them obtained spiral shocks.

Only in recent years, Yukawa, Boffin & Matsuda (1997) showed that shock waves are obtained by the SPH method when the number of particles is increased so that the resolution is enhanced. They obtained spiral shocks with the binary having a mass ratio of 1 and with a specific heat ratio, γ, of 1.2, although they did not with γ of 1.1 or 1.01. Thereafter, Lanzafame & Belvedere (1997, 1998), Boffin, Haraguchi & Matsuda (1999) obtained basically similar results.

With respect to three-dimensional calculation by the finite difference/volume method, Sawada & Matsuda (1992) performed the calculation with use of the TVD method and generalized curvilinear coordinates, and obtained spiral shocks for $\gamma = 1.2$. They calculated, however, only up to half the orbital rotation period, and hence it is still doubtful if the generation of shock waves is an established phenomenon or merely a transient one. Our group therefore conducted a series of three-dimensional simulations as described in the following section (see Makita & Matsuda 1999, Matsuda, Makita & Boffin 1999).

Bisikalo et al. have performed a series of three-dimensional calculations similar to ours (1997a, b, 1998a, b and c). Their study is very much like ours and therefore particularly worth commenting. They used the TVD method and Cartesian coordinates. The region of calculation was $[-a, 2a] \times [-a, a] \times [0, a]$, where a is the separation, and the region was divided with a non-uniform lattice of $78 \times 60 \times 35$ or $84 \times 65 \times 33$. (In our calculation given later, the region is $[-a, a] \times [-0.5a, 0.5a] \times [0, 0.5a]$, which is divided into $200 \times 100 \times 50$.) They used the equation of state of a perfect gas and calculated with the specific heat ratio $\gamma = 1.01, 1.2$. They calculated over a sufficiently long period of 12-20 orbital periods. One of the main points concluded by them is the absence of “hot spot”, i.e. high-temperature region generated on collision of the gas inflowing from the L1 point with the accretion disk. As described later, our results also support this conclusion. They also conclude that no accretion disk forms with $\gamma = 1.2$, while our results show the formation. In addition, they mention, as the cause for the generation of spiral shocks, rather than the tidal force of the companion, the interaction between the L1 flow and an expanded atmosphere, to which we do not agree.

3. Method of calculation

3.1. Basic assumptions

We consider a mass-accreting star (main star) with mass $M1$ and a mass-losing star (companion) with mass $M2$. The mass ratio is limited to $q = M2/M1 = 1$ only in the present work. The companion is assumed to have filled the critical Roche lobe. The motion of the gas flow having blown out from the surface of the companion is calculated by solving the time-dependent Euler equations.

The basic equations governing the motion of the gas are the Euler equations with no viscous term. We consider only numerical viscosity which is incorporated into calculation from the scheme employed, and consider neither molecular viscosity nor α viscosity. We further assume that the equation of state of the gas is expressed by that of a perfect gas and take the specific heat ratios 1.01 and 1.2. With accretion disks, radiative cooling plays an important role. Judging from the present-day computer power, it is, however, very difficult to incorporate the effect of the radiative cooling into three-dimensional calculation. We therefore try to simulate the gas cooling to some extent, by using a small specific heat ratio γ.

The calculation method used is the Simplified Flux Vector Splitting (SFS) method (Jyounouchi et al. 1993, Shima & Jyounouchi 1994, 1997). A MUSCL-type approach is used, with the calculation accuracy being of 2nd order for both time and space.

With the centers of the main star and the companion being at $(0, 0, 0)$ and $(-1, 0, 0)$, respectively, the region of calculation is $[-1.5, 0.5] \times [-0.5, 0.5] \times [0, 0.5]$, where the lengths are scaled by the separation a. The region is divided by grid points of $201 \times 101 \times 51$. The main star is represented by a hole of $3 \times 3 \times 2$, while the companion by a surface along the critical Roche lobe.

3.2. Initial conditions and boundary conditions

3.2.1. *Initial condition*

The entire space except the companion is filled with a gas having a density 10^{-7} and a sonic velocity of $c_0 = 0.02$. Here the density is, as described later, is normalized by that of the gas on the surface of the companion. The sonic velocity value is sufficiently smaller than that in Makita, Miyawaki & Matsuda (1998), who assumed the density and the sonic velocity to be 10^{-5} and $10^{1/2}$, respectively. In their case the gas initially placed in the region of calculation has a low density, but a high temperature. These values are not so important in two-dimensional calculations, since the initial gas will be removed from the region of the accretion disk eventually.

In three-dimensional calculations, however, the initial gas will partially remain in a space above the disk. In the inner regions where the disk is thin, the disk gas and the initial gas may mix with each other numerically, thereby increasing the disk temperature artificially. As a result, inner spiral arms may wind-in more loosely than in the actual case. In order to prevent this, we assume the density and temperature of the initial gas to be at sufficiently low levels.

3.2.2. *Boundary conditions*

The inside of the hole representing the main star is always filled with the above-described initial gas. The gas having reached the vicinity of the main star therefore accretes due to the low pressure inside the star. The outer boundary is also fixed at these values. At the outer boundary, the artificial reflection of waves is suppressed to a minimum, so that the calculation can proceed stably.

The inside of the companion is assumed to be always filled with a gas having a density of 1 and a sonic velocity of 0.02. The inside of the companion is assumed to be free from the gravity. The pressure difference between the inside and outside of the companion surface causes the inside gas to flow out, the mass flux of which is evaluated by solving the corresponding Riemann problem.

4. Results of calculation

4.1. Density distribution and spiral shocks

Figures 2 show the density distribution on the orbital plane at $t = 72$, for γ =1.01 and 1.2, respectively. Figures 3 show the iso-density surfaces for the same cases. The L1 flow, i.e. a flow coming out of the L1 point, penetrates into the inside of the accretion disk and does not slow down on collision with the disk or form a "hot spot". This is discussed in detail later. Another point to be noted is that, while a nearly circular accretion disk and a pair of spiral shocks are observed for $\gamma = 1.01$, the shape of the accretion disk is considerably deformed for $\gamma = 1.2$. These results differ from the preceding results obtained by Makita et al. (1998), who observed a clearly circular accretion disk and spiral shocks for both $\gamma = 1.01, 1.2$. Our calculations differ from those of Makita et

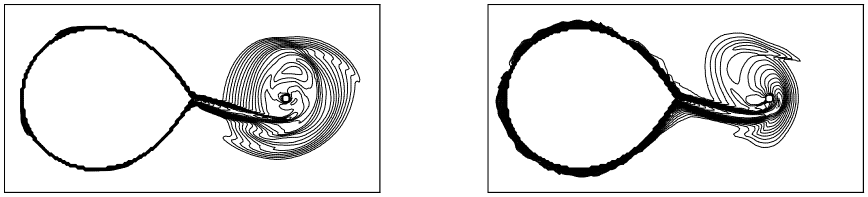

Figure 2. Density contours in logarithmic scale of the gas on the rotational plane at $t = 72$. Left: the case for $\gamma = 1.01$. Right: the case for $\gamma = 1.2$.

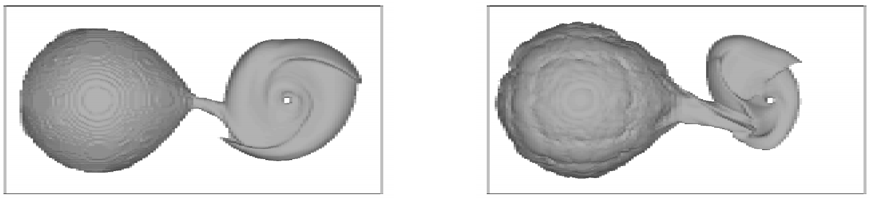

Figure 3. Iso-density surface at $log \rho = -4.2$. Left: the case for $\gamma = 1.01$. Right: the case for $\gamma = 1.2$.

al. in the size of calculation region, the mechanism of L1 flow formation, and the density and temperature of initial gas. Of these, the former two may not influence much, and the differences in the conditions of initial gas may have caused the difference of the results. Bisikalo et al. (1998a, b, c) argue that, for $\gamma = 1.2$, a considerably large part of the gas flowing in through the L1 point will flow out of the region, so that no accretion disk can be formed. Our results stand in-between those of Makita et al. and Bisikalo et al. The question of knowing who is right remains to be answered.

4.2. Penetration of L1 flow into accretion disk

It is difficult to visualize a velocity field, which is a vector field. We use the Line Integral Convolution (LIC) method to visualize the velocity field. For LIC, see Cabral & Leedom (1993). The details of the visualization will be given in a separate paper (Nagae, Fujiwara, Makita, Hayashi & Matsuda, in preparation). Figure 4a shows the stream lines on the whole rotational plane. Figure 4b shows the iso-density surface of $\log \rho = -4.0$ with stream lines on it. The stream lines on the rotational plane are also shown. In Figs. 4c and d a portion of the space between the L1 point and the main star has been expanded, so that one can easily see how the L1 flow penetrates into the accretion disk. As seen from Figures 4, the L1 flow coming smoothly through the L1 point does, without slowing down on encounter with the disk, penetrate into the inside of the disk. Figure 4a shows that the rotational flow inside

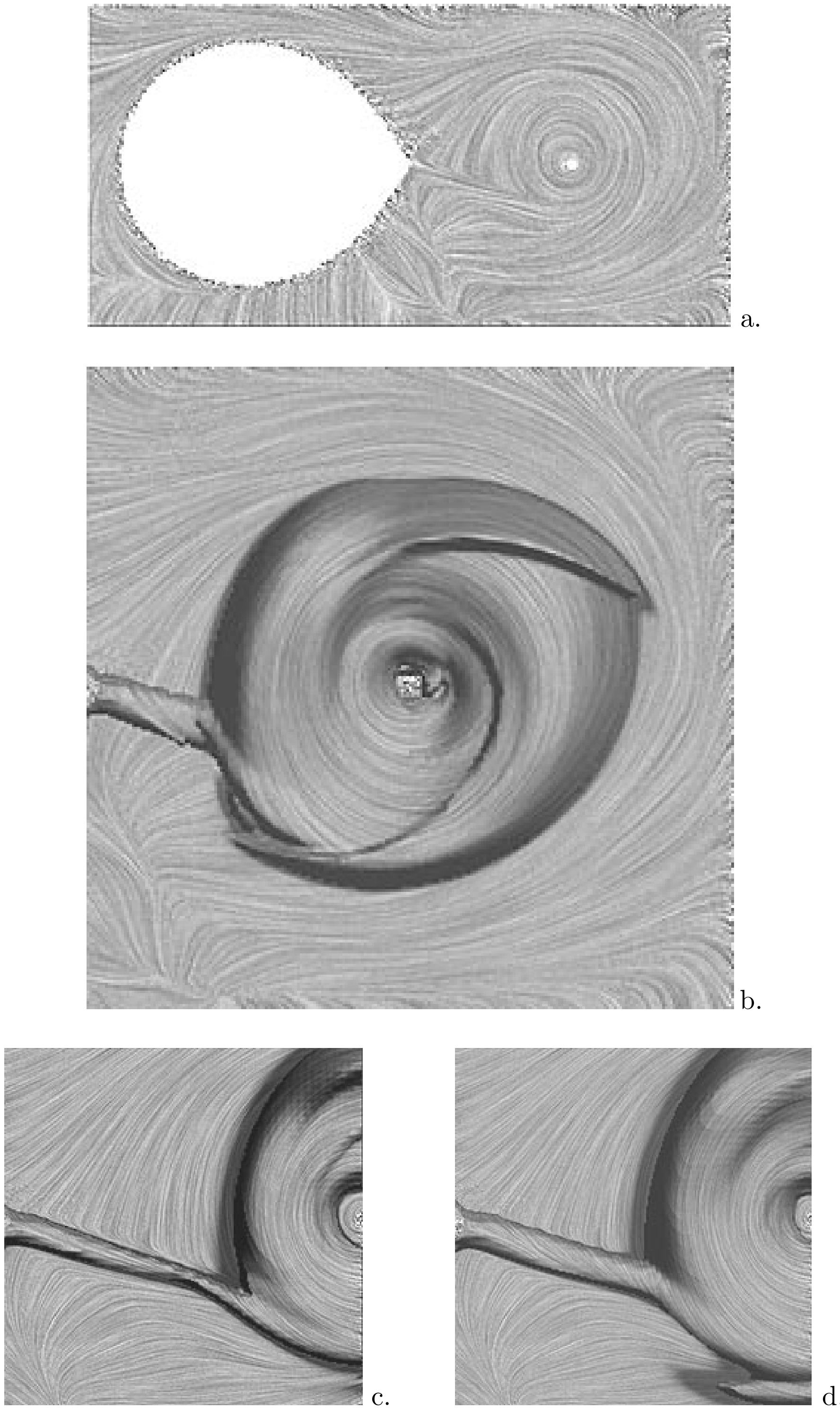

Figure 4. Iso-density surface and flow lines on it for the case of $\gamma = 1.01$ at $t = 72$. a) Flow lines on the whole rotational plane. b) Flow lines on the iso-density surface of $\log \rho = -4.0$ about the mass-accreting star. c) Blow up of L1 stream with $\log \rho = -2.8$, d) Same as c except $\log \rho = -3.3$.

the disk changes, on the orbital plane, its direction rapidly on collision with the L1 flow. Figures 4b, c, d show that the gas placed a little above the orbital plane, along the z-axis, gets over the L1 flow on collision therewith. That is, the L1 flow looks like a bar inserted into the accretion disk, and the disk flow collides with the L1 flow to form a bow shock on the upstream side (upper side in the Figure). Figure 4b shows that the iso-density surface swells along a spiral arm. Figure 4b shows an iso-density surface with the lowest density, where one can see that the L1 flow penetrates the accretion disk, like a spear.

Bath et al. (1983) studied the penetration of the L1 flow into the accretion disk. Whether or not the L1 flow penetrates into the disk depends on the relative magnitude of density between the L1 flow and the disk. An L1 flow having a larger density will penetrate, while one having a smaller density will not. Chochol et al. (1984) argued that such a penetration of the L1 stream was observed in a symbiotic star CI Cyg.

5. Comparison with observations

Eleven years after the numerical discovery of spiral shocks by Sawada et al. (1986), Steeghs, Harlaftis & Horne (1997), using a method called "Doppler Tomography" for observation, discovered the presence of spiral structure on an accretion disk of IP Pegasi for the first time. Since then, spiral structures have been found successively in other accretion disks : SS Cyg (Steeghs et al. 1996), V347 Pup (Still, Buckley & Garlick 1998), EX Dra (Spruit in preparation).

The Doppler tomography method comprises at first observing the time history of the emission line spectrum of hydrogen or helium for 1 orbital period, on a binary system close to edge-on. The method then makes a distribution map of the emission lines on the velocity space (V_x, V_y) by an ingenious technique, the maximum entropy technique. The obtained map is called Doppler map and basically corresponds to a hodograph in hydrodynamics. Presence of a spiral distribution in a Doppler map corresponds to a spiral distribution of emission lines in the physical (configuration) space. The shape of the physical-space brightness distribution, however, cannot be derived from the corresponding Doppler map.

On the other hand, numerical simulations can give the physical-space distribution of a physical quantity, from which a Doppler map can be prepared. Obtaining a distribution of emission lines of an element in the real space needs both the knowledge of temperature and complex radiative transfer calculations, and is not easy. We therefore prepare a

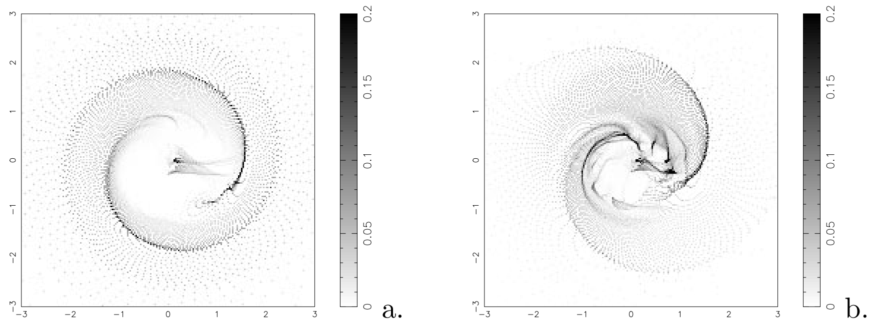

Figure 5. Doppler map prepared based on two-dimensional calculation; a): $\gamma = 1.01$ b): $\gamma = 1.2$

Doppler chart, which maps the density distribution, not the emission line distribution, on the velocity space. This procedure may be deemed valid from expectation that a high-density region be of high brightness.

Figures 5a, b are Doppler maps, prepared in the above-described manner, for γ =1.01 and 1.2, and show the presence of widely opened spiral structures. Matsuda, Makita, Yukawa & Boffin (1999) and Makita, Yukawa, Matsuda & Boffin (1999) performed two-dimensional SPH calculations and prepared Doppler maps. The maps thus prepared agree well with those based on our two-dimensional finite-difference calculations, and also very well with the observations.

This agreement is remarkable, since the results of these two-dimensional calculations show that, in particular for $\gamma = 1.2$, the obtained Mach number is as small as less than 10, which means that the disk has a considerably high temperature. On the other hand, observations expect much lower temperatures and a Mach number of 20-30 for the accretion disks of cataclysmic variables. For such a high Mach number case, numerical simulation would give spiral shocks with tightly winding, so that no Doppler map agreeing with observations can be prepared (Godon, Livio & Lubow 1998).

Steeghs et al. (1997) observed spiral structures in the outburst phases of a cataclysmic variable, but not in the quiescent phases. This suggests that only with the disk having a high temperature, spiral shocks winding to such modest angle as to be observable can be formed. It is hard to prepare a correct, meaningful Doppler map based on the results of three-dimensional calculations, since the distribution observed is that of the emission component at an optical depth of 1, i.e. on the photosphere of the accretion disk. The preparation thus requires complex calculations of radiative transfer, which is beyond the scope of the present paper. One point of interest here is that the three-dimensional calculations with a small $\gamma = 1.01$ have given moderately loosely winding spiral shocks, which means that even with accretion disks with comparatively low temperature the winding-in shape tends to agree with observations.

6. Summary

1. Spiral shock waves on accretion discs in close binary systems were found numericaly by Sawada et al. (1986a, b, 1987) in their two-dimensional hydrodynamic finite difference simulation.

2. Spiral shock waves are generated by an oval deformation of the gravitational potential and the Lindblad resonances associating with it. Spiral shocks are seen in galactic disks and the primordial solar nebula as well as accretion disks.

3. Three-dimensional simulations also exhibit the presence of spiral shocks despite of some counterarguments.

4. The stream from L1 point penetrates into the accretion disk rather than being blocked by it. Therefore, so-called hot spot is not formed in our present case, which fact agrees with the results by Bisikalo et al. (1997a, b, 1998a, b, c).

5. Spiral structure was found in the accretion disk of a dwarf nova IP Pegasi by Steeghs et al. (1997) using Doppler tomography technique.

6. Theoretical Doppler maps are made based on two-dimensional simulations, and they agree well with observed ones despite of high temperature of gas in the simulation.

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